# Advanced optimal control: from reusable rocket landing to efficient training of neural networks

IPSA course 2026

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Laboratoire des Signaux et Systèmes
CNRS and Université Paris-Saclay



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1. Examination date: Mai 5, 2026 from 1:30pm to 5:30pm.
2. Examination kind: Per group, oral presentation (slides) + questions on the course.
3. Examination duration: 10 minutes presentation + 15/20 minutes questions (board).

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   b. 10 points max. 10/15 minutes of questions on the course at the blackboard.
      Anything in Lecture 1 + Lecture 2. ONLY LECTURES ARE ALLOWED!

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- Consider this Optimal Control Problem (OCP) - Minimum time 2D car model:

$$\inf_{u \in L^\infty([0,T],[-1,1])} T$$
$$\dot{x}(t) = \cos\theta(t),\ x(0) = 0,\ x(T) = x_1,$$
$$\dot{y}(t) = \sin\theta(t),\ y(0) = 0,\ y(T) = y_1,$$
$$\dot{\theta}(t) = ku(t),\ \theta(0) = 0,\ \theta(T) = \theta_1, \qquad k = \text{constant} > 0,\ x_1, y_1, \theta_1 \in \mathbb{R} \text{ given.}$$

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- What to do:
  a. Assuming $(p_x, p_y) \neq 0$, prove that optimal controls are in the bang-bang form:

$$u(t) = \begin{cases} 1, & p_\theta(t) > 0, \\ -1, & p_\theta(t) < 0, \\ 0, & p_\theta(t) = 0, \end{cases} \qquad \text{where } p_\theta \text{ is the } \theta \text{ component of the adjoint vector.}$$

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  b. Numerically solve OCP, with whatever method you prefer, showing optimal 2D trajectories and controls-in-time for several values of $k > 0,\ x_1, y_1, \theta_1 \in \mathbb{R}$.

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- Slide 4: Present the numerical method you chose to solve OCP in all its details, e.g., numerical variables, ODE discretization, cost and constraints, solver, etc.
- Slide 5: Show and explain the plots of the solution to your numerical method; these should include 2D trajectories and control-in-time for different $k > 0,\ x_1, y_1, \theta_1 \in \mathbb{R}$.

# What files do you send to Riccardo Bonalli?

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EACH GROUP SEND ONLY 1 JUPYTER NOTEBOOK AND 1 PRESENTATION
Other, late, or non conforming files will make YOU LOSE MANY POINTS!

# Presentation order

1. Group 1 - 1:30pm to 2:00pm
2. Group 2 - 2:00pm to 2:30pm
3. Group 3 - 2:30pm to 3:00pm
4. Group 4 - 3:00pm to 3:30pm
5. Group 5 - 3:30pm to 4:00pm
6. Group 6 - 4:00pm to 4:30pm
7. Group 7 - 4:30pm to 5:00pm
8. Group 8 - 5:00pm to 5:30pm

**Questions?**

**Otherwise:**
**Best of luck!!!**